# Assessing the Accuracy of Geodetic Measurements for the VLBI2010 Observing Network

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**Abstract.** We investigate the expected accuracy of geodetic estimates made by the next generation VLBI2010 network. To do this we simulated the effect of several known input contributions including troposphere turbulence, troposphere mapping function error, antenna deformation, and site pressure error. These contributions propagate to estimates of station coordinates. By comparing estimated values of parameters with known input values, we can evaluate biases that result from mismodeling.

**Keywords.** VLBI2010, Reference frames

## 1 Introduction

In previous work, we have investigated the expected precision of the VLBI2010 observing network (4). In this studies, we looked at the effects of troposphere, clock, and measurement noise by performing Monte Carlo simulations. Here we try to answer the question: What is the level of systematic error for VLBI2010? One of the goals is to generate an error budget. Here, we consider the following errors: 1) troposphere turbulence, 2) clock error, 3) observation noise, 4) hydrostatic troposphere mapping function error, 5) antenna gravitational deformation, and 6) site pressure error.

## 2 Simulation description

For these simulations, we used the same 16-station network shown in Figure 1 that was used by the VLBI2010 working group. The observing schedule was a uniform sky schedule with each antenna observing at 60 observations/hour (T. Searle and B. Petrachenko, personal communication). The concept of the uniform sky schedule is that a series of pairs of approximately diametrically opposed quasars is observed during an observing session. For each pair, antennas on one half of Earth observe one quasar and the remaining antennas on the other side of the Earth observe the other quasar. The scheduling software chooses a sequence of source pairs to maximize the uniformity of the sky distribution of sources at each of the antennas. This type of schedule can work because the antennas in the network were assumed to slew with the same speed. For current networks with mixed types of antennas, this strategy does not work ideally. The 60 observations/hour schedule corresponds to an antenna with a slew rate of 5 deg/sec in azimuth and 1.2 deg/sec in elevation.

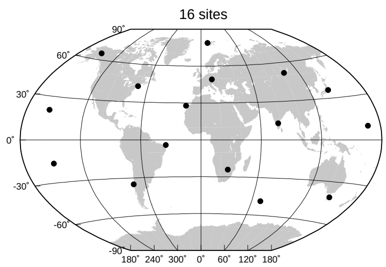


**Figure 1.** Global 16-site simulation network.

Clock delays for each station are modeled as a random walk plus an integrated random walk corresponding to an Allan variance of $1\text{x}10^{-14}$ at 50 minutes. A white noise contribution corresponding to the observation uncertainty is added. The wet delay error is based on Kolmogorov turbulence delay modeling. The term $\tau_{si}$ refers to systematic errors that are studied in simulations. The observation error model is:

$$O - C = [m_{wet}(el_2)\tau_{wz2} + clk_2 + \tau_{s2}]$$
$$-[\mathrm{m}_{wet}(el_2)\tau_{wz1} + clk_1 + \tau_{s1}] + \sigma_{obs}$$

## 3 Modeling Results

In the following sections, we discuss the effect of each error source separately on the topocentric site position estimates. We summarize the

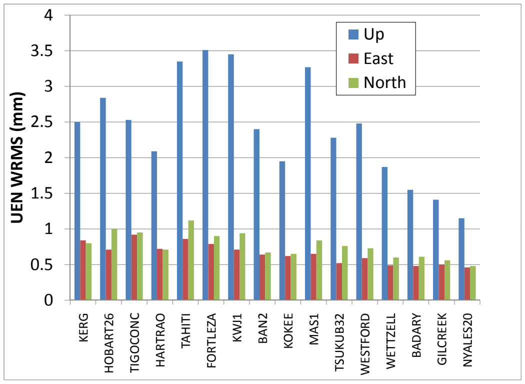


**Figure 2.** Site position (UEN) scatter due to tropopsheric turbulence, where sites are ordered by increasing latitude

results at the end in the error budget shown in Table 1.

### 3.1 Turbulence

We derived a latitude and site-height dependent model for the troposphere refractive index structure constant $C_n$, where we have averaged over the seasonal variation. We started with the $C_n$ and heights computed by T. Nilsson (personal communication) from a global distribution of high resolution radiosonde site data. The $C_n$ increase towards the equator corresponding to increased troposphere water vapor content. The delay model is based on the Treuhaft-Lanyi Kolmogorov turbulence model. To test the model we ran simulations with the two-week series of CONT05 data. Baseline length repeatability scatter from the simulation runs is reasonably close to observed scatter, overestimating the observed scatter by a factor of 1.2± 0.4.

The main systematic effect shown in Figure 2 from the turbulence error is the increase in vertical scatter from about 1 mm at high latitude sites like Ny Alesund to 3 mm near the equator. There is some variability due to site height, for example, scatter is reduced for sites with large heights - Kokee (1177 m), HartRao (1435 m), and BAN2 (835 m). The horizontal scatter is 0.5-1.0 mm and is nearly independent of latitude. Since turbulence is a noise process there is essentially no bias produced in the geodetic estimates.

### 3.2 Clock Error

We modeled the effect of clock error characterized it by an Allan variance of 1x$10^{-14}$ at 50 minutes. The resulting site vertical scatter is 0.5-0.8 mm and horizontal scatter is only 0.2 mm. There is minimal latitude dependence. There is essentially no resulting bias. This level of clock variance is fairly conservative in the sense of what we expect to have for the VLBI2010 system.

### 3.3 Observation Error

The nominal precision of the VLBI2010 observable is 4 psec. Modeling this as a white noise process in a simulation yields a vertical precision of only 0.15 mm and horizontal precision of 0.05 mm. The simulation is linear in the sense that if we have a 12 psec observable, the site position scatter is tripled.

### 3.4 Hydrostatic Mapping Function

Currently, the best available mapping functions are the VMF1, which are based on one-dimensional raytracing of ECMWF weather model profiles (1). VMF1 assumes that the troposphere about a site is azimuthally symetric. Comparisons have been made between VMF1 and one-dimensional raytracing of radiosonde profile data. Niell (5) computed the WRMS delay error of the hydrostatic VMF1 at a 5 deg elevation angle for a globally-distributed set of radiosonde data sites. We simulated this error as an error in the a-coefficient of the continued fraction form of the mapping function. The error is linear in the derivative of the mapping function with respect to the a-coefficient. Similarly, we simulated the bias error of the mapping function using bias errors from radiosonde delay comparisons from Böhm et al. (2). The site vertical scatter increases from 0.5 mm near the equator to 2 mm at high latitude in Figure 3. It is seen in Figure 4 that vertical bias error has a magnitude as large as 0.8 mm at high latitude and is positive in the northern hemisphere and negative in the southern hemisphere. There is also a systematic bias error of the North component of site position. This is due to the no-net-translation constraint applied in the solution and the fact that there are more Northern hemisphere sites for the 16-site network. The problem is difficult to avoid because the land mass in the Southern

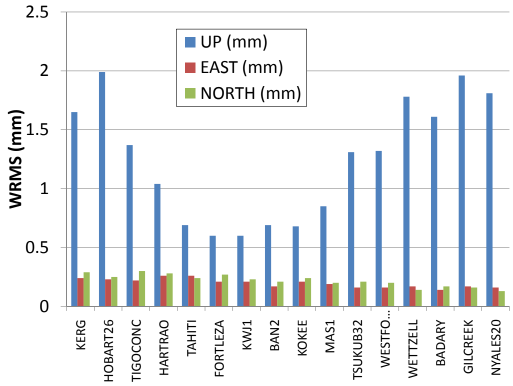


**Figure 3.** Site position (UEN) scatter due to VMF1 mapping function error, where sites are ordered by latitude.

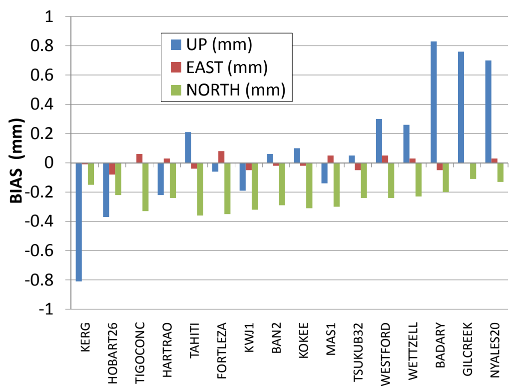


**Figure 4.** Site position (UEN) bias due to VMF1 mapping function bias.

hemisphere is so much less than in the Northern hemisphere.

### 3.5 Site Pressure Data

It is essential that pressure be estimated accurately at each site. To assure this, 1) pressure sensor calibration must be maintained, 2) pressure data cannot be missing, and 3) one must account for sensor height relative to the reference point of the VLBI antenna. To quantify the effect of pressure error, we simulated the effect of a 10 mbar pressure error. Figure 5 shows that this error biases vertical estimates by 0.15-0.20 mm/mbar for a 5 deg minimum elevation cutoff. The error decreases by a factor of about 2 between a 5 and 10 deg cutoff.

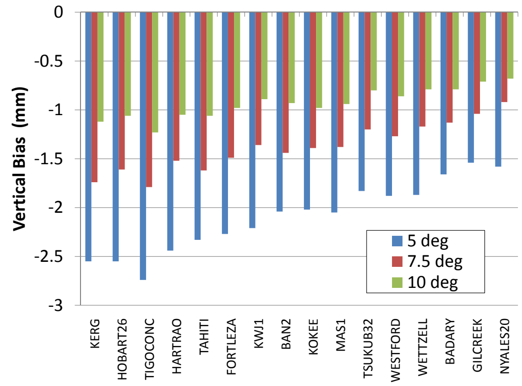


**Figure 5.** Site position (UEN) bias due bias error site pressure of 10 mbar.

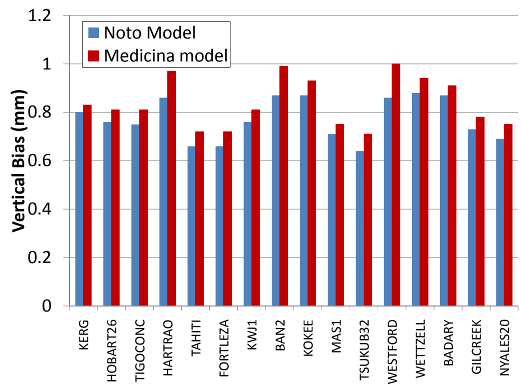


**Figure 6.** Site vertical bias error due to gravitational deformation.

### 3.6 Antenna Gravitational Deformation

Gravitational deformation is an important effect that until recently has not been included in VLBI analysis. Sarti and Abondanza (6) have measured the deformation as function of elevation of the 32-meter antennas at Noto and Medicina using a laser scanner to determine the focal point variation, terrestrial survey to measure the receiver position variation, and finite element modeling to determine the vertex position variation. They used a model for the signal path dependence on elevation caused by deformation based on the work of Clark and Thomsen (3). The model is linear in the variations of focal length, receiver position, and vertex position. The effect for Noto is less because of improvements made to its surface. The models for these antennas were scaled down from 32 m to the 12 m nominal diameter of VLBI2010 antennas, assuming that the

**Table 1.** Site Vertical Position Error Budget

| Parameter | Bias (mm) | WRMS (mm) |
|---|---|---|
| Turbulence | $< 0.5$ | 1-3 |
| Hydrostatic mapping | 0.5-1.5 | 0.5-2.0 |
| Clock error | $< 0.2$ | 0.6 |
| Gravity deform | 0.6-1.0 | - |
| Obs noise (4 ps) | $< 0.1$ | $< 0.15$ |
| Thermal (mm/C) | 0.07 | - |
| Pressure (mm/mb) | 0.15-0.25 | - |
| Source structure | ? | ? |

effect is proportional to the area of the antenna. Simulation results in Figure 6 show that the deformation causes a station vertical bias of 0.7-1.0 mm.

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