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To: ECGVM Workshop
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Subject: Baseline length repeatability as evaluator

1. Objective

Present some ideas for using baseline length repeatability as a measure of the quality of geodetic VLBI data and as a measure of improvement as a result of modeling, analysis, or other change.

2. Some definitions

A. Evaluating the accuracy of baseline length uncertainties

R_L = baseline length repeatability
= weighted RMS residual to a model, e.g. residual to $L = L_0 + (dL/dt)*t$

s_{L0} = uncertainty in R_L
= weighted mean formal baseline length error
= $\mathrm{sqrt}(\sum(1/\sigma)/ \sum(1/\sigma^2))$

D_L = modeling error, i.e. how much larger scatter in data is than expected from formal error
= $\mathrm{sign}(R_L^2 - s_{L0}^2)(\mathrm{sqrt}(\mathrm{abs}(R_L^2 - s_{L0}^2))$
To be useful, there should be an uncertainty associated with the derived value of D_L in order to judge whether the modeling error is significant. I haven't found a satisfactory way to calculate this.

B. Similar expressions can be used to evaluate the change in some aspect of the data analysis.

R_{Li} = baseline length repeatability for model *i*, *i* = 1, 2.

d_L = difference in WRMS residuals between models
= $\mathrm{sign}(R_{L2}^2 - R_{L1}^2)(\mathrm{sqrt}(\mathrm{abs}(R_{L2}^2 - R_{L1}^2))$

C. If the uncertainties of the positions of all antennas are considered in terms of σ_v and σ_h in the vertical and horizontal, respectively, then the expected baseline length scatter can be represented as

$R_L^2 = 2*[(1 - (L/2R_e)^2) \bullet \sigma_h^2 + (L/2R_e)^2 \bullet \sigma_v^2]$

Fitting a function of this form to the R_L then gives estimates of the average horizontal and vertical uncertainties.

Equivalently, the length repeatability, R_L^2, can be fit as

$R_L^2 = a \bullet L^2 + b$.

The equivalent horizontal and vertical uncertainties are then given by

$\sigma_h = \mathrm{sqrt}(b/2)$

$\sigma_v = \mathrm{sqrt}(2R_e^2 a + b/2)$

The uncertainties in σ_h and σ_v are found by substituting the corresponding diagonal covariance terms (from the least_square fit) for a and b.

For the case that a model change is expected to affect primarily the height of a station, differences of R_{Li}^2 can then be view as changes in σ_v. See example in the next section.

3. Example – unmodeled error in terms of horizontal and vertical error

I have used data through 2003 (the last time I got baseline repeatability numbers from Goddard) to illustrate the unmodeled error in a solution. the results are shown in figure 0. This is similar to the paper of MacMillan and Ma (1994). The estimates and uncertainties for the horizontal and vertical uncertainties are shown in the Figure 0.

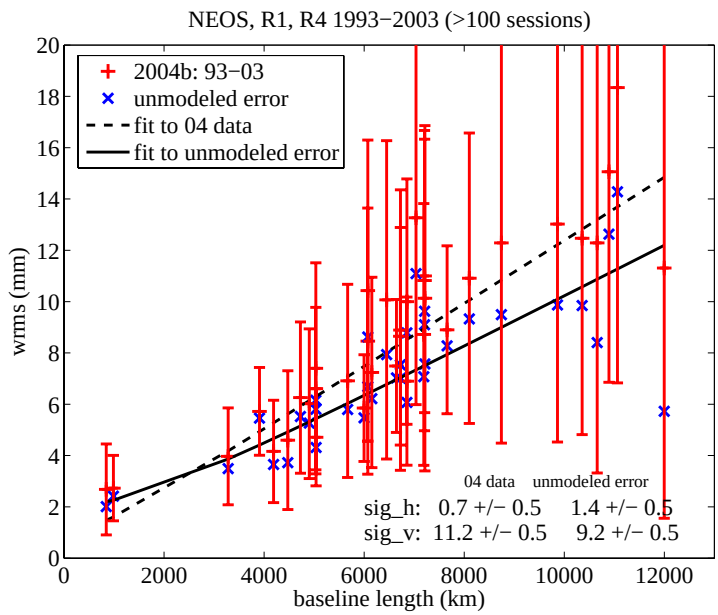

Figure 0. Baseline repeatability and unmodeled error.

Of course values derived this way are un underestimate of the unmodeled error since the baseline length uncertainties for each twenty-four session include added delay noise in the process of achieving a chi-square/degree of freedom of 1.

4. Example – evaluating the change in a model

Changing the mapping function should primarily affect estimates of the vertical component of position, which can be related to baseline length as given in 2.C.

The errors in site height (right axis) for NMFh and IMFh are shown as a function of latitude in Figure 1:

At mid-latitude the difference is about 4 mm. Thus in changing from NMFh to IMFh the baseline length repeatability would be expected to be reduced in a quadratic sense (if the atmosphere error is independent of the other errors leading to baseline scatter) by

$$d_L = \text{sign}(R_L(NMF)^2 - R_L(IMF)^2) \bullet (\text{sqrt}(\text{abs}(R_L(NMF)^2 - R_L(IMF)^2))$$

The repeatabilities for the CONT94 baselines are shown in Figure 2, and the quadratic differences are shown in Figure 3.

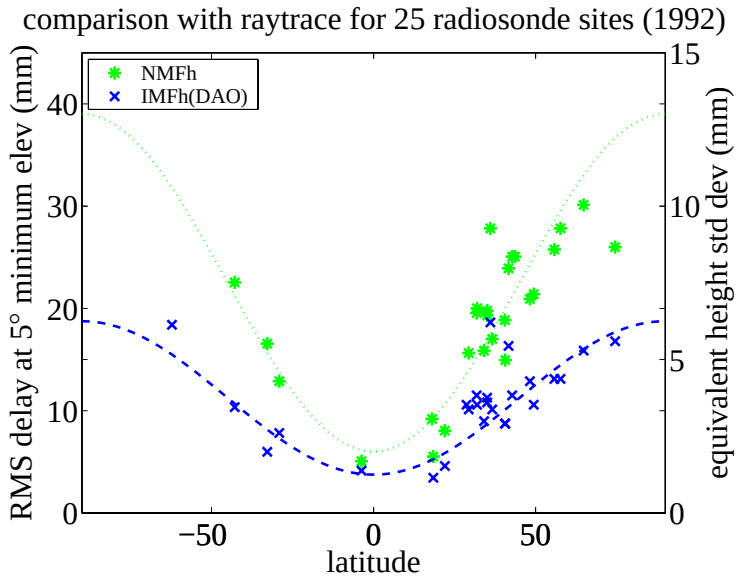

Figure 1. Equivalent daily height error (right axis) from hydrostatic mapping functions.

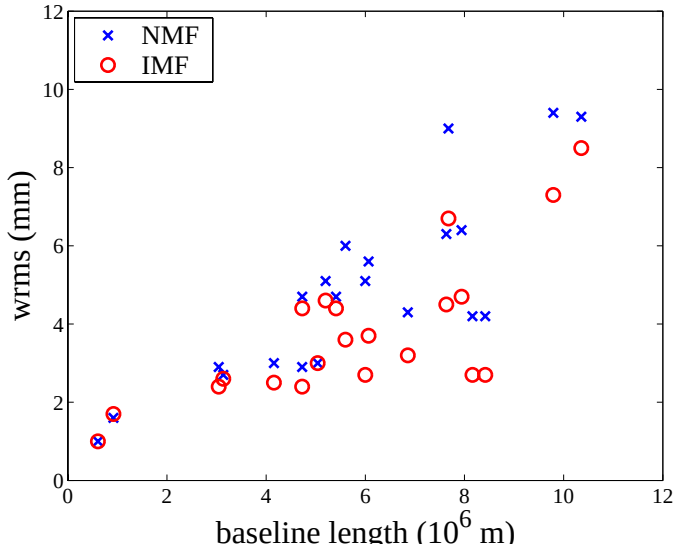

Figure 2. Repeatabilities of CONT94 baselines when analyzed with NMFh and with IMFh.

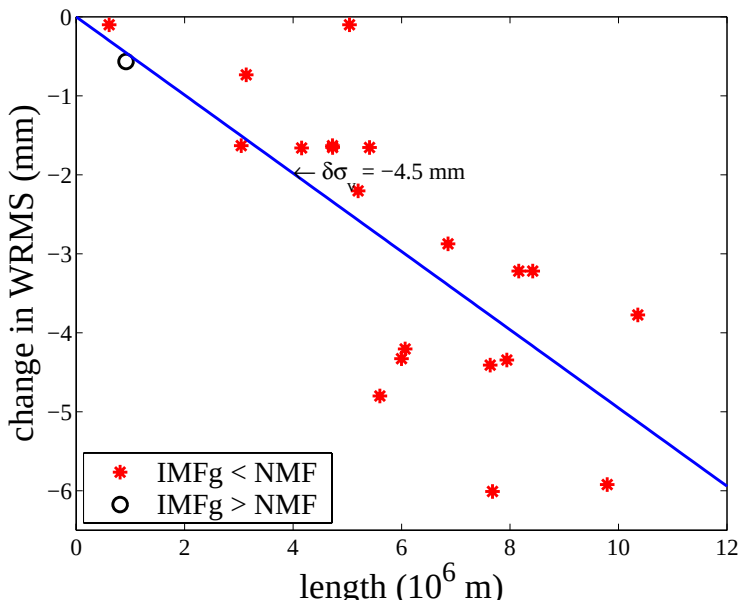

Figure 3. Quadratic difference of repeatabilities of CONT94 baselines when analyzed with NMFh and with IMFh. The sign convention corresponds to reducing the contribution of vertical error by using IMFh instead of NMFh.

In this case the change in repeatability appears consistent with that expected from the model change. However, I do not know what to use for uncertainties on the points in Figure 3 in order to be able to assess quantitatively the level of consistency.

5. Comments

This note is intended only to illustrate ways to use baseline length repeatability to evaluate the modeling. The characteristics of the variance, whether noise-like or having harmonic variations (periodic terms such as annual), must also be investigated.